Q1

1

Find an expression for $\frac{dy}{dx}$ in terms of x and y for the following

(i) $2ye^x + 5x^2y^2 = 8$
(ii) $3x\tan y = 2x^2$

[4]

i) $\frac{d}{dx}$ $\quad 2ye^x + e^x 2\frac{dy}{dx} + 5x^2 2y\frac{dy}{dx} + y^2 10x = 0$

$$\frac{dy}{dx}\left(\cancel{2}e^x + \cancel{10}^{5}x^2y\right) = -\cancel{2}ye^x - \cancel{10}^{5}xy^2$$

$$\boxed{\frac{dy}{dx} = \frac{-ye^x - 5xy^2}{e^x + 5x^2y}}$$

ii) $\frac{d}{dx}$ $\quad 3x(\sec^2 y)\frac{dy}{dx} + (\tan y)3 = 4x$

$$\frac{dy}{dx}\left(3x\sec^2 y\right) = 4x - 3\tan y$$

$$\boxed{\frac{dy}{dx} = \frac{4x - 3\tan y}{3x\sec^2 y}}$$

Q2a

2a

(a) Given that

$$y^2 + 4x^2 - e^y = 0$$

find the positive value of x when $y = 0$

[1]

(b) Hence, or otherwise, find the value of the gradient of

$$y^2 + 4x^2 - e^y = 0$$

at the point where $y = 0$ and x is positive.

[4]

a) Sub $y=0$ into eqn.

$$0^2 + 4x^2 - e^0 = 0$$

$$4x^2 = 1$$

$$x^2 = \frac{1}{4}$$

$$x = \sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

Choose the positive value of x.

$$\boxed{x = \frac{1}{2}}$$

Q2b

2b

(a) Given that

$$y^2 + 4x^2 - e^y = 0$$

find the positive value of x when $y = 0$

$x = \frac{1}{2}$

[1]

(b) Hence, or otherwise, find the value of the gradient of

$$y^2 + 4x^2 - e^y = 0$$

at the point where $y = 0$ and x is positive.

[4]

b) $\frac{d}{dx}$ $\quad 2y\frac{dy}{dx} + 8x - e^y\frac{dy}{dx} = 0$

Sub in $x = \frac{1}{2}$, $y = 0$

$\cancel{2(0)\frac{dy}{dx}} + 8\left(\frac{1}{2}\right) - e^0\frac{dy}{dx} = 0$

$4 - \frac{dy}{dx} = 0$

$\boxed{\frac{dy}{dx} = 4}$

Q3a

3a

The curve C has equation $2xy^2 - x^2 = 16$
Line L has equation $x = 4$

(a) Show that the two points where C intersects L have equal gradients.

[5]

(b) What else can you deduce about the two points where C and L intercept?

[1]

a) At $x = 4$

$2(4)y^2 - 4^2 = 16$

$8y^2 = 32$

$y^2 = \frac{32}{8} = 4$

$y = \sqrt{4} = \pm 2$ So the points of intersection are: $(4, 2)$ $(4, -2)$

$\frac{d}{dx}$ $\quad 2x\, 2y\frac{dy}{dx} + y^2 2 - 2x = 0$

$\left(\frac{dy}{dx}\right)(4xy) = 2x - 2y^2$

$\frac{dy}{dx} = \frac{2x - 2y^2}{4xy}$

At $(4, 2)$:
$\frac{dy}{dx} = \frac{2(4) - 2(2)^2}{4(4)(2)} = \boxed{0}$

At $(4, -2)$:
$\frac{dy}{dx} = \frac{2(4) - 2(-2)^2}{4(4)(-2)} = \boxed{0}$

$\boxed{\therefore \text{ the two gradients are equal.}}$

Q3b

3b

The curve C has equation $2xy^2 - x^2 = 16$
Line L has equation $x = 4$

(a) Show that the two points where C intersects L have equal gradients.

The two gradients are 0.

[5]

(b) What else can you deduce about the two points where C and L intercept?

[1]

b) Since the gradients are zero at $(4,2)$ and $(4,-2)$, these must be stationary points.

Q4

4

Verify that the point $(-1, 0)$ lies on the curve with equation

$$3xe^y + 2x + 5 = 4y$$

and find the equation of the tangent to the curve at the point $(-1, 0)$.
Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

[5]

$x = -1 \quad y = 0$

LHS: $3(-1)e^0 + 2(-1) + 5 = -3 - 2 + 5 = 0$

RHS: $4(0) = 0$

LHS = RHS
$\therefore (-1, 0)$ lies on curve

$y - \underset{0}{y_1} = m(x - \underset{-1}{x_1})$

$\frac{d}{dx}$ $\quad 3x(e^y)\frac{dy}{dx} + e^y 3 + 2 = 4\frac{dy}{dx}$

Sub in $x = -1$, $y = 0$

$3(-1)e^0\frac{dy}{dx} + e^0 3 + 2 = 4\frac{dy}{dx}$

$5 = 7\frac{dy}{dx}$

$m = \frac{dy}{dx} = \frac{5}{7}$

$y - 0 = \frac{5}{7}(x - (-1))$

$7y = 5x + 5$

$\boxed{7y - 5x - 5 = 0}$

Q5a

5a

(a) Show that the derivative function of the curve given by

$$\ln y - 2xy^3 = 8$$

is given by

$$\frac{dy}{dx} = \frac{2y^4}{1 - 6xy^3}$$

[5]

(b) Find the equation of the normal to the curve given in part (a) at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

[3]

a) $\frac{1}{y}\frac{dy}{dx} - \left(2x\, 3y^2\frac{dy}{dx} + y^3 2\right) = 0$

$$\frac{dy}{dx}\left(\frac{1}{y} - 6xy^2\right) = 2y^3$$

$$\frac{dy}{dx} = \frac{2y^3}{\frac{1}{y} - 6xy^2} \times \frac{y}{y}$$

$$\boxed{\frac{dy}{dx} = \frac{2y^4}{1 - 6xy^3}}$$

Q5b

5b

(a) Show that the derivative function of the curve given by

$$\ln y - 2xy^3 = 8$$

is given by

$$m_t = \frac{dy}{dx} = \frac{2y^4}{1 - 6xy^3}$$

[5]

(b) Find the equation of the normal to the curve given in part (a) at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

[3]

b) $y - y_1 = m_n(x - x_1)$

$y = 1$

$\cancel{\ln(1)} - 2x1^3 = 8$

$x = \frac{8}{-2} = -4$ $(-4, 1)$

$$m_t = \frac{dy}{dx} = \frac{2(1)^4}{1 - 6(-4)(1^3)} = \frac{2}{25}$$

$$m_n = \frac{1}{\frac{dy}{dx}} = \frac{-1}{\frac{2}{25}} = \frac{-25}{2}$$

$$y - 1 = -\frac{25}{2}(x - (-4))$$

$$2y - 2 = -25x - 100$$

$$\boxed{25x + 2y + 98 = 0}$$

Q6

6

Show that the stationary points on the curve with equation

$$xy^2 - 4x^2 = 64$$

occur when $x = 4$, and find the **exact** y-coordinates of the stationary points.

[6]

$\frac{d}{dx}$ $x2y\frac{dy}{dx} + y^2 - 8x = 0$

At stationary points, $\frac{dy}{dx} = 0$

$\therefore y^2 - 8x = 0$

$y^2 = 8x$

Sub in $y^2 = 8x$ into eqn for curve

$x(8x) - 4x^2 = 64$

$4x^2 = 64$

$x^2 = 16$

$x = \pm 4$

When $x = 4$ $\quad y^2 = 8(4) \quad y = \pm 4\sqrt{2}$

When $x = -4$ $\quad y^2 = 8(-4) \quad y = \sqrt{-32}$ no real solutions!

$\therefore$ Stationary points: $\boxed{(4, 4\sqrt{2}),\ (4, -4\sqrt{2})}$

Q7a

7a

(a) Verify that the point $A(1, 1)$ lies on the curve with equation

$$\ln(xy) + xy^2 = 1$$

[1]

(b) The tangent at point A intercepts the x-axis at point B and the y-axis at point C. Find the area of the triangle OBC.

[8]

a) LHS $= \ln(1 \times 1) + 1 \times 1^2 = 1$

RHS $= 1$

LHS = RHS

$\therefore A(1, 1)$ lies on the curve.

Q7b

7b

(a) Verify that the point $A(1, 1)$ lies on the curve with equation

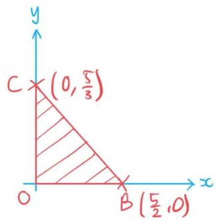
$$\boxed{\ln(xy)} + xy^2 = 1$$

$\ln x + \ln y$

[1]

(b) The tangent at point A intercepts the x-axis at point B and the y-axis at point C. Find the area of the triangle OBC.

[8]



b) $y - y_1 = m(x - x_1)$

Find gradient, m, of tangent

$\frac{d}{dx}$ $\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} + x\,2y\frac{dy}{dx} + y^2 = 0$

$\frac{1}{1} + \frac{1}{1}\frac{dy}{dx} + 1\times2\times1\times\frac{dy}{dx} + 1^2 = 0$

$3\frac{dy}{dx} + 2 = 0$

$\frac{dy}{dx} = -\frac{2}{3} = m$

Find eqn of tangent using

$y_1 = 1$, $x_1 = 1$ and $m = -\frac{2}{3}$

$y - 1 = -\frac{2}{3}(x-1) = -\frac{2}{3}x + \frac{2}{3}$

Tangent eqn: $y = -\frac{2}{3}x + \frac{5}{3}$

At B, $y = 0$

$0 = -\frac{2}{3}x + \frac{5}{3}$

$x = \frac{5}{2}$

At C, $x = 0$

$y = -\frac{2}{3}(0) + \frac{5}{3}$

$y = \frac{5}{3}$

Area OBC $= \frac{1}{2}bh = \frac{1}{2}\left(\frac{5}{2}\right)\left(\frac{5}{3}\right) = \boxed{\frac{25}{12}}$ square units

Q8

8

Show that

$$\frac{d}{dx}[a^{kx}] = ka^{kx}\ln a$$

where a and k are constants.

[3]

Let $y = a^{kx}$

$\ln y = \ln a^{kx}$

$\ln y = kx \ln a$

$\frac{1}{y}\frac{dy}{dx} = k\ln a$

$\frac{dy}{dx} = yk\ln a$

Sub in $y = a^{kx}$

$\boxed{\frac{d(a^{kx})}{dx} = ka^{kx}\ln a}$